Versatile Mathematics

COMMON MATHEMATICAL APPLICATIONS

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Chapter

7

Set Theory

The Library of Congress, the national library of the United States, contains a vast collection of works that fill over 800 miles of bookshelves, and somewhere around 10,000 new works are added each day. The question naturally arises: how can anything be found in such a huge, diverse collection? The answer, of course, lies in categorization, or organization.

Librarians, among others (like grocery store planners, for instance), have to be experts at categorization, in order to arrange their collections in such a way that items are easy to find. The basics of this skill are natural, though; you have an intuitive idea of how to categorize objects in a way that makes sense. When we categorize, what we're really doing is creating **sets**. For instance, a library has a fiction section, where the set of novels in their collection are placed. Within that set of novels, there may be a **subset** of young adult fiction, a subset of historical fiction, and so on. As a student, you can be categorized by your major, the classes you're taking, your year in school, etc., each of which can be expressed as a set.

It turns out that much of higher mathematics (which we don't do in this book) uses the terms and concepts of set theory extensively. We'll only see the basic structure of set theory in this chapter, but this way of thinking is valuable to those who study mathematics in more detail.

In fact, if you compare this chapter to the chapter on logic, you'll notice some similar ideas coming up, which illustrates the ties that set theory has to other areas of mathematics.

SECTION 7.1 Basic Concepts

The definition of a set is a very simple one:

Definition: A **set** is a collection of objects.

The fact that this definition is so simple is important; the simplicity is what allows us to apply the ideas of set theory to so many different fields, because we only have to be working with "objects."[1]

As far as notation, we use curly braces to enclose the objects in a set, called the **elements** of the set, and we separate the elements with commas. Thus, the set that consists of the numbers 1, 2, and 3 would be written

$$S = \{1, 2, 3\}.$$

Also, based on the definition above, all it takes to define a set is to describe what objects are in it, and **the order in which they are listed is irrelevant**. Thus, the following two sets are identical:

$$A = \{a, b, c\} \quad \text{and} \quad B = \{b, c, a\}$$

This way of describing a set by listing its elements is often called **roster notation**.

EXAMPLE 1 SET NOTATION

Let S be the set of the days of the week. Write this using roster notation.

Solution

We could list the days of the week in any order, but of course, there is a traditional order to them.

$$\boxed{S = \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}}\}$$

EXAMPLE 2 SET NOTATION

Create a set that represents three courses that a college student may be taking.

Solution

In this case, there is no natural order. To make an example, we'll choose the following three courses:

$$\boxed{S = \{\text{Introduction to Philosophy, Western Civilizations, Statistics}\}}$$

TRY IT

Let S be the set of the first five odd numbers. Write S using roster notation.

Since a set is defined solely by what elements belong to it, we need a way to describe whether something belongs to a specific set or not. We get a new symbol to do this:

"Is an element of"

The symbol $\in$ is read "belongs to" or "is an element of."

Ex: $a \in \{a, b, c\}$ can be read "a is an element of the set $\{a, b, c\}$" or "a belongs to the set $\{a, b, c\}$"

Putting a stroke through it changes the meaning to "does not belong to."

Ex: $d \notin \{a, b, c\}$

[1] In fact, the objects in a set could be sets themselves, so we can construct a set of sets, and so on.

USING ELEMENT NOTATION — EXAMPLE 3

Place $\in$ or $\notin$ in each of the following blanks to make each statement true.

(a) Apple ______ F, where F is the set of all fruits

(b) g ______ $\{a, e, i, o, u\}$

(c) 3 ______ the set of positive real numbers

Solution

It should be clear that an apple belongs to the first set, and 3 belongs to the set of positive real numbers, but g does not belong to the given set of letters.

(a) Apple $\boxed{\in}$ F, where F is the set of all fruits

(b) g $\boxed{\notin}$ $\{a, e, i, o, u\}$

(c) 3 $\boxed{\in}$ the set of positive real numbers

Common Sets

There are some sets that come up more than others, so we'll take a moment here to list them, along with a symbol that we can use for each.

$\mathbb{R}$	The set of all real numbers; basically, any number that you can use to count or measure something[2]
$\mathbb{Z}$	The integers[3,4]: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
$\mathbb{N}$	The natural numbers (or counting numbers)[5]: $\{1, 2, 3, \ldots\}$

These symbols, along with the element symbol, give us another way to define sets, using what is sometimes called **set-builder notation**, where instead of listing the elements of a set, we give a rule that defines them.

For instance, we could write

$$S = \{x \in \mathbb{N} \mid x \text{ is even}\}.$$

If we wanted to read it, we would need to know that the vertical line in the middle of the set is read "such that," so the line above would read

"S is the set of all x in the natural numbers **such that** x is even"

and if we started listing elements of this set, we would write

$$S = \{2, 4, 6, 8, 10, \ldots\}.$$

Notice that we have to use ellipses when we write it this way, to denote that this set keeps on going and going. By writing it in set-builder notation, though, we have a concise way of accounting for all the elements of the set by giving the rule that they all have to satisfy.

[2]Notice that we can't list this set like we do the others, because there's no way to account for all of them.
[3]The letter Z comes from the German word for "number."
[4]The ellipses (...) indicate that this is an infinite set, extending forever in both directions.
[5]These are the positive integers.

We do this naturally in language; we might talk about the set of all sophomores at a college, or the set of people on the Dean's List. Remember, to define a set, all we need is a clear indication of what elements belong to a set; we can do this by listing them, or by giving a rule that they all must follow.

EXAMPLE 4 SET BUILDER NOTATION

List the elements of the following sets that are described using set builder notation. If the sets are infinite, list the first five elements.

(a) $A = \{x \in \mathbb{Z} \mid x \geq -3\}$

(b) $B = \{x \in \mathbb{N} \mid 2 \leq x \leq 8\}$

Solution

(a) $A = \{x \in \mathbb{Z} \mid x \geq -3\}$

$$\boxed{A = \{-3, -2, -1, 0, 1, \ldots\}}$$

(b) $B = \{x \in \mathbb{N} \mid 2 \leq x \leq 8\}$

$$\boxed{B = \{2, 3, 4, 5, 6, 7, 8\}}$$

TRY IT

Find a way to write the following set in set builder notation.

$$C = \{\ldots, 3, 4, 5, 6, 7\}$$

Subsets

The idea of a subset is an intuitive one; if I told you that your class is a subset of students at your college, you would intuitively understand what I meant. We can make this definition more precise, though:

Subset

We say that one set A is a **subset** of another set B, written

$$A \subseteq B$$

if every element in A is also an element of B.

This notation can be informally read as "A is contained in B," which can be helpful in remembering the notation, since the subset symbol looks like a capital C.

In other words, your class is a subset of students at your college because every student in your class is also a student at your college. However, we probably couldn't say that your friends form a subset of students at your college, because, while many of your friends may be at your school, you may have friends that do not belong to that set.

SUBSETS **EXAMPLE 5**

Place $\subseteq$ or $\not\subseteq$ in each of the following blanks to make each statement true.

(a) {Spring, Fall} ______ {Winter, Spring, Summer, Fall}

(b) {Green, Blue, Red} ______ {Red, Yellow, Green}

(c) {North, South, East, West} ______ {West, South, North, East}

(d) $\{-2, 5, -1\}$ ______ $\mathbb{Z}$

(e) $\{-2, 5, -1\}$ ______ $\{x \in \mathbb{Z} \mid x > 0\}$

Solution

(a) {Spring, Fall} $\boxed{\subseteq}$ {Winter, Spring, Summer, Fall}, because both elements in the first set also appear in the second set.

(b) {Green, Blue, Red} $\boxed{\not\subseteq}$ {Red, Yellow, Green}, because the element "Blue" appears in the first set, but not the second one.

(c) {North, South, East, West} $\boxed{\subseteq}$ {West, South, North, East}, because, once again, every element in the first set also appears in the second set (they happen to be the exact same set, just written in a different order).

(d) $\{-2, 5, -1\}$ $\boxed{\subseteq}$ $\mathbb{Z}$, because each of those numbers in the first set are integers.

(e) $\{-2, 5, -1\}$ $\boxed{\not\subseteq}$ $\{x \in \mathbb{Z} \mid x > 0\}$, because not all of the numbers in the first set meet the condition in the second set (two of them are negative).

TRY IT

Place $\subseteq$ or $\not\subseteq$ in each of the following blanks to make each statement true.

(a) {1,7,6,3} ______ $\{x \in \mathbb{N} | 1 \leq x \leq 10\}$

(b) {a, b, c} ______ {a, e, i, o, u}

Notice in the third item of the example above that the given set was contained in itself. This will always be true by definition: naturally, every element in A will be an element in A, so

$$A \subseteq A.$$

That is why we include a line below the C, similar to how we use the symbol $\leq$ to indicate "less than **or** equal to." The subset symbol can be thought of as "is contained in **or** equal to."

Therefore, we can also define a **proper subset**, which is a subset that is **not** equal to the set that contains it. This is written without the line underneath the C.

For instance, $\{1, 2, 3\}$ is a proper subset of $\{1, 2, 3, 4\}$, but $\{1, 2, 3, 4\}$ is not. In general,

$$A \subset B$$

if A is completely contained in B, and B has at least one extra element that A doesn't have.

EXAMPLE 6 PROPER SUBSETS

Place $\subset$ or $\not\subset$ in each of the following blanks to make each statement true.

(a) $\{0, 1, 2\}$ ______ $\mathbb{Z}$

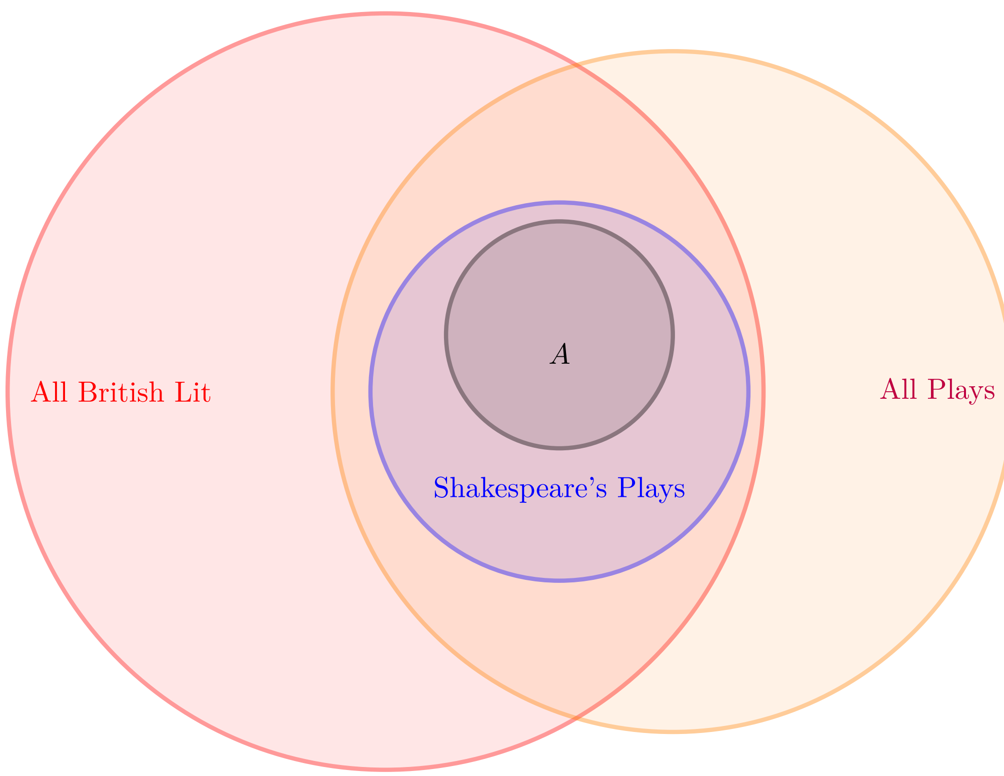
(b) $\{0, 1, 2\}$ ______ $\{x \in \mathbb{Z} \mid 0 \leq x \leq 2\}$

Solution

(a) $\{0, 1, 2\}$ $\boxed{\subset}$ $\mathbb{Z}$, because every element in the first set belongs to the set of integers, $\mathbb{Z}$.

(b) $\{0, 1, 2\}$ $\boxed{\not\subset}$ $\{x \in \mathbb{Z} \mid 0 \leq x \leq 2\}$, because the two sets are equal, so the first set is not a *proper* subset of the second.

Of course, a given set could be a subset (or proper subset) of many different sets. For instance, consider the set $A = \{$"Much Ado About Nothing", "MacBeth", "A Midsummer's Night Dream"$\}$. This is a subset of the set of William Shakespeare's plays, which of course is itself a subset of all plays and all British literature, as shown in the **Venn diagram** below (we'll see more of these diagrams in the next section).



$A = \{$"Much Ado About Nothing", "MacBeth", "A Midsummer's Night Dream"$\}$

The Empty Set

What if we describe a set like the following?

$$\{x \in \mathbb{R} \mid x < 2 \text{ and } x > 5\}$$

If you try to list the elements in this set, you'll quickly find that there are none, because it is impossible for a number to be simultaneously less than 2 and greater than 5.

This is an example of one way to describe the **empty set**[6], whose name gives it away: the empty set is defined as the set with no elements.

The Empty Set

The set $\{\ \}$, which contains no elements, is called the **empty set** and is written with the symbol $\varnothing$.

(note: it is not written $\{\varnothing\}$, because that would mean a set with one element, and that element is the empty set)

EMPTY SET — EXAMPLE 7

Which of the following sets are empty?

(a) The set of the days of the week whose names start with P.

(b) The set of students at Frederick Community College under the age of 25.

Solution

(a) This set is empty; since there are no days of the week that start with P, there are no elements in this set.

(b) Since there are students that fall into this category, this set is not empty.

Cardinality

The **cardinality** of a set is just a fancy term for the number of elements it contains.

Cardinality

The **cardinality** of a set A is the number of *distinct* elements in A.

The cardinality of A is denoted $n(A)$, or sometimes $|A|$.

CARDINALITY — EXAMPLE 8

Find the cardinality of each of the following sets.

(a) $A = \{3, 5, 9, 32\}$

(b) $B = \{2, 3, 2, 4, 2\}$

(c) $C = \{31, 32, \ldots, 58\}$

(d) $D = \varnothing$

[6] Note that we call it *the* empty set because every empty set is identical to every other empty set, so there's really just one distinct empty set.

Solution

(a) Since there are four elements listed, $\boxed{n(A) = 4}$.

(b) There are five elements listed, but notice that 2 is listed three times, so there are really only three *distinct* elements in this set. Therefore, $\boxed{n(B) = 3}$.

(c) Even though only three elements are shown, the ellipses indicate that this set includes all the integers from 31 to 58. Since this includes 28 integers (note carefully, not 27), $\boxed{n(C) = 28}$.

(d) There are no elements in the empty set, so $\boxed{n(D) = 0}$.

Summary

Since we've encountered so many new terms and symbols in this section, it may be helpful to pause for a moment and review these new concepts.

New Terms

Set A collection of objects

Subset A is a subset of B if all the elements of A are also elements of B

Proper subset A subset that is not equal to its containing set

Empty set The set that contains no elements

Cardinality The number of distinct elements in a set

New Symbols and Notation

Set notation Uses curly braces: $\{\ldots\}$

$\in$ Is an element of

$\mathbb{R}$ The set of real numbers

$\mathbb{Z}$ The set of integers

$\mathbb{N}$ The set of natural numbers (integers starting at 1)

$\subseteq$ Subset

$\subset$ Proper subset

$\varnothing$ The empty set

$n(A)$ The cardinality of A

Sidenote: Infinite Sets

For some sets, we can count the number of elements and call that number their cardinality. These are called **finite sets**. Others, like the integers, for instance, are **infinite sets**. Can we talk about the cardinality of an infinite set?

It turns out that we can. Georg Cantor, the founder of set theory, was one of the first mathematicians to dive headfirst into an investigation of infinite sets (for which his career suffered tremendously as others ridiculed his work at the time), and his contributions have since been lauded as groundbreaking.

Let's start with the natural numbers ($\mathbb{N} = \{1, 2, 3, \ldots\}$), an infinite set. The cardinality of this set is not a number, because there are infinitely many elements in the set, but we call this cardinality aleph-nought (aleph being the first character in the Hebrew alphabet), written $\aleph_0$.

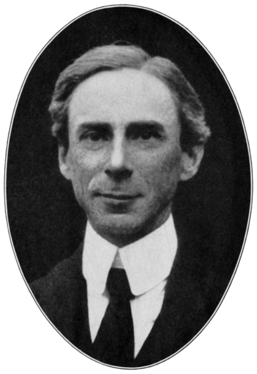
Now, here comes the mind-bendy part, so hang on tight. It makes sense to say that two sets have the same number of elements (the same cardinality) if we can place their elements in one-to-one correspondence. In other words, if each Shark can pair up with exactly one Jet, there must be the same number of Sharks and Jets.

You may not believe this the first time you hear it, but it's true: the natural numbers and the integers have the same cardinality. But hold on, there are clearly more integers, right, since there are negative numbers in the integers? But look, we can place the natural numbers and the integers in a one-to-one correspondence:

$\mathbb{N}$	1	2	3	4	5	6	7	8	9	10	$\ldots$
	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	
$\mathbb{Z}$	0	1	-1	2	-2	3	-3	4	-4	5	$\ldots$

If you think that's **at all** interesting, Google "Hilbert's Hotel" and enjoy.

Any infinite set that is infinite in this same way, meaning that it can be placed in one-to-one correspondence with the natural numbers, is called a **countable** set. There are some infinite sets that are uncountable; the set of real numbers is the notable example of this. There is no way to count through the real numbers and make any progress, unlike the way that we can count through the natural numbers or the integers.



Bertrand Russell in 1916

Russell's Paradox

Since the definition of a set is so broad, we can in theory define a set of sets. For instance, you could treat every baseball team in the MLB as a set, where the elements are the players on that team, and then the MLB could be a set of sets, where its elements are the teams in the league.

However, a subtle problem arises here, and Russell's Paradox, discovered by Bertrand Russell in 1901, is a famous illustration of it.

Before we state this paradox, we'll give two informal representations of it.

Barber Paradox Suppose in a certain town, there is a barber who shaves all men who don't shave themselves, and these are the only men he shaves. Does he shave himself?

Of course, if he doesn't shave himself, he is one of the ones in the category of people that he does shave, and if he does shave himself, he is not, leading to a contradiction.

Library Paradox Consider another situation: there are 100 libraries in a country, and each library has a book that is a written catalog of all the books in that library. As each library is compiling its catalog, they face this question: should they list this catalog as one of the books in the library? Fifty of the libraries choose to list their catalog *in* their catalog, and 50 choose not to.

Now, suppose the national library wants to create a master catalog of all these individual catalogs. In fact, they create two lists: one is the list of all the catalogs that list themselves, and the other is the list of those that don't.
Here comes the paradox: should this master catalog list itself in the category of those that don't? If it lists itself in that category, it no longer belongs to that category, and vice versa.

Russell's Paradox Russell's Paradox, in precise terms, goes like this: let R be the set of all sets that are not members of themselves. If R is a member of itself, then by definition, it cannot be a member of itself, and vice versa. Symbolically, we could write it this way:

$$R = \{x \mid x \notin x\} \text{ implies } R \in R \text{ if and only if } R \notin R$$

Solution This paradox may seem abstract and a little contrived, but it was a powerful blow to the development of set theory, because it struck at the heart of what it means to define a set.

To resolve this paradox, the generally accepted set theory (Zermelo-Fraenkel) essentially did away with the ability to define sets the way that Russell did, requiring sets to be **well-defined**.

Exercises 7.1

In problems 1–4, write each set using roster notation.

1. $A =$ the set of letters in the word "Mississippi."

2. $B =$ the set of the four seasons in a year.

3. $C =$ the set of natural numbers less than 6.

4. $D =$ the set of even natural numbers between 7 and 13.

In problems 5–13, fill in the blank with either $\in$ or $\notin$ to make each statement true.

5. 6 ____$\{2, 4, 6, 8, 10\}$

6. 7 ____$\{2, 4, \ldots, 12, 14\}$

7. 11 ____$\{1, 2, 3, \ldots, 9\}$

8. 37 ____$\{1, 2, 3, \ldots, 50\}$

9. 3 ____$\{x \in \mathbb{Z} \mid x > -2\}$

10. -2 ____$\mathbb{N}$

11. 20 ____$\{x \in \mathbb{N} \mid 12 \leq x < 20\}$

12. 14 ____$\{x \in \mathbb{Z} \mid x < 100\}$

13. -3 ____$\{x \in \mathbb{Z} \mid x \geq -8\}$

In problems 14–17, write each set using roster notation.

14. $\{x \in \mathbb{N} \mid x < 10 \text{ and } x \text{ is odd}\}$.

15. $\{x \in \mathbb{N} \mid 2 \leq x \leq 5 \text{ and } x \text{ is even}\}$.

16. $\{x \in \mathbb{N} \mid x < 4\}$.

17. $\{x \in \mathbb{N} \mid 3 < x < 7\}$.

In problems 18–23, fill in the blank with either $\subseteq$ or $\not\subseteq$ to make each statement true.

18. $\{1, 3, 6\}$ __$\{1, 2, 3, 4\}$

19. $\{2, 4, 6\}$ __$\{1, 2, \ldots, 10\}$

20. $\{-1, 0, 2\}$ __$\mathbb{N}$

21. $\varnothing$ __$\mathbb{N}$

22. $\{b, c, d, e\}$ ____$\{a, b, c, d, e, f, g, h\}$

23. $\{x \mid x \text{ is a cat}\}$ ____$\{x \mid x \text{ is a black cat}\}$

In problems 24–29, fill in the blank with either $\subset$ or $\not\subset$ to make each statement true.

24. $\{x, y, z\}$ ____$\{x, u, w, v, z, y, t\}$

25. $\{0, 3, 4, 7, 1\}$ ____$\{1, 3, 0, 7, 4\}$

26. $\varnothing$ ____$\{a, b, c, d\}$

27. $\{x \mid x \text{ is a woman}\}$ ___ $\{x \mid x \text{ is a person}\}$

28. $\{x \in \mathbb{N} \mid 5 < x < 12\}$ ____the set of natural numbers between 5 and 12

29. $\{\}$ ____$\varnothing$

In problems 30–32, determine whether each set is empty or not.

30. $A = \{0\}$

31. $B = \{x \mid x$ is a month of the year whose name begins with the letter X$\}$

32. $C = \{x \mid x < 2 \text{ and } x > 7\}$

In problems 33–38, find the cardinality of each set.

33. $A = \{12, 14, 16, 18, 20\}$

34. $B = \{1, 3, 5, \ldots, 25\}$

35. $C = \{x \in \mathbb{N} \mid 3 \leq x < 14\}$

36. $D = \{x \in \mathbb{N} \mid x < 2 \text{ and } x \geq 5\}$

37. $E = \varnothing$

38. $F = \{x \mid x$ is a letter in the word "elephant"$\}$

SECTION 7.2 Set Operations

Once we can talk about individual sets, we can start thinking about relationships among these sets. For instance, think back to the example from the chapter introduction about a library categorizing books in their collection.

If they have a set of books written in the 20th century, and they have a set of science fiction books, we could "combine" these two sets and look for only the science fiction novels written in the 20th century, excluding other books written in the 20th century and other science fiction.

On the other hand, we could look for any book written in the 20th century that is NOT in the category of science fiction. And of course there are other combinations we could make with these two sets.

This is similar to the way we can use a search engine like Google to refine the results of a search.[7]

We'll define four operations that we can use to describe the relationship between two sets:

1. The **complement** of a set
2. The **union** of two sets
3. The **intersection** of two sets
4. The **difference** between two sets

You'll find that these operations are well named, because their names describe what they do.

Before we get to those, though, we need to define what we call the **universal set**, because we'll need to keep track of this universal set when we look at examples.

Universal Set

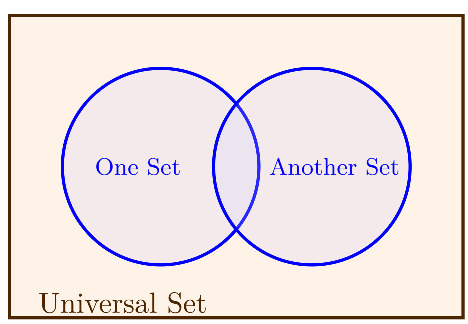
In a particular situation,[a] the **universal set** is the set of all the objects that we are considering in this context.

[a]The universal set will be different in different problems.

In other words, if the problems starts by describing the books in a library, then the set of all the books at that library will be the universal set. On the other hand, if the problem starts with the students in your classroom, those students will form the universal set.

Venn Diagrams

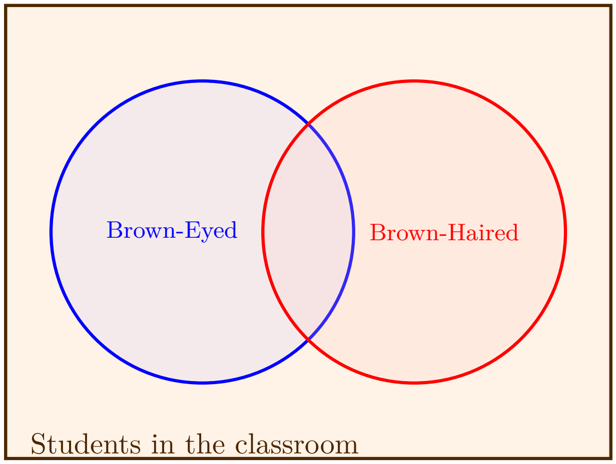
As we go through these operations, it will be helpful to use diagrams to visualize them; these diagrams are called **Venn diagrams**, since they were introduced by John Venn in 1880. A basic Venn diagram consists of a box (usually shown) that represents the universal set, and circles inside that box that represent sets that we want to talk about.



Notice the overlapping region in the middle; this represents the elements that belong to both sets.

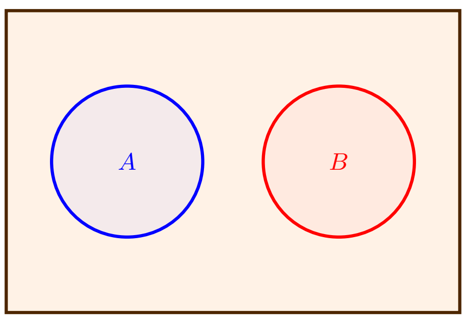
[7]See the first section of the chapter on Logic for more details.

For instance, we could categorize students in your classroom by hair color and eye color, specifically looking at the set of students who have brown hair and the set of students with brown eyes.



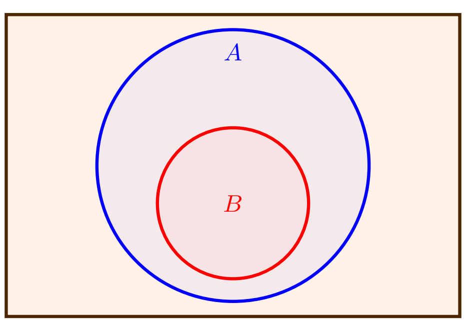
There are (probably) some students in the overlapping region, who have brown hair and brown eyes. But it is certainly possible to have brown eyes and not brown hair, and vice versa. Also, it is possible to not have brown eyes or brown hair, and these would be the students inside the rectangle but outside of both circles.

We could have an example, though, where there are no shared elements between the two sets. For instance, if the two sets we considered were "letters in the Greek alphabet" and "letters in the Hebrew alphabet," those two circles would not overlap at all. We call these **disjoint sets**.



Disjoint Sets

We could also have an example where one set is completely contained in another, like in the case of the sets "all students" and "students at your school."



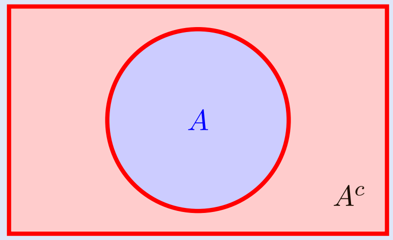
Proper Subset

Now that we have Venn diagrams at our disposal, we're ready to define the complement, union, intersection, and difference.

Complement

Complement

The **complement** of a set A, denoted[a] A^c consists of all elements (in the universal set) that **do not** belong to A.



$$A^c = \{x \mid x \in U \text{ and } x \notin A\}$$

[a]Other books might write A' or $\overline{A}$ to indicate the complement.

EXAMPLE 1 **COMPLEMENT**

In the alphabet, find the complement of V, the set of vowels.

Solution
We're assuming that y is not a vowel

The complement of the vowels (i.e. the letters that aren't vowels) is the set of consonants:

$$\boxed{V^c = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}}$$

Notice that we especially need a well defined universal set (like the 26 letters of the alphabet) to be able to describe the complement of a set clearly.

EXAMPLE 2 **COMPLEMENT**

Given the universal set $U = \mathbb{N}$, find the complement of

$$A = \{x \in \mathbb{N} \mid x \geq 10\}.$$

Solution

The complement of the natural numbers greater than or equal to 10 is, as you may expect, the natural numbers less than 10 (notice that 10 is in A, so it is not in A^c):

$$\boxed{A^c = \{x \in \mathbb{N} \mid x < 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}}$$

EXAMPLE 3 **COMPLEMENT**

If the universe is $U = \{1, 2, 3, \ldots, 10\}$, find the complement of the set $E = \varnothing$.

Solution

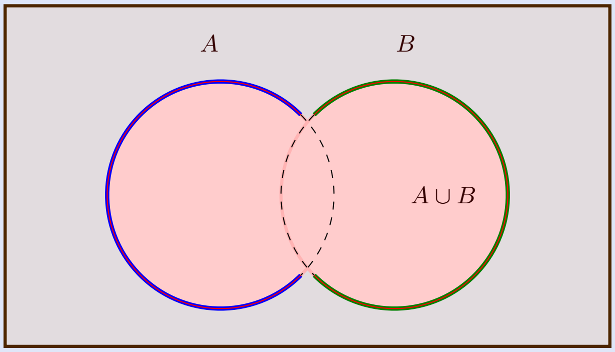
The complement of the empty set is the set of all points in the universe:

$$\boxed{E^c = U = \{1, 2, 3, \ldots, 10\}}$$

Union

Union

The **union** of two sets A and B, denoted[a] $A \cup B$, consists of all elements that are **either in** A **or in** B **OR BOTH**.



$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

[a]You can remember this because the symbol looks like a U for union.

This is the part that trips up the most people. The union includes elements that are in both sets, which isn't necessarily how we tend to use the word "or"; for instance, if someone said "write down your first name or your last name," you wouldn't think to write both. The way we often use it in English is called the *exclusive* OR, meaning that it doesn't include "...or both," but in set theory (and logic) we use the *inclusive* OR.

Again, note that all it takes for an element to be in the union of two sets is for it to belong to *at least* one of them. For instance, you'd be in the union of "people who own a car" and "people who own a bike" as long as you owned *at least* one of those; if you owned a car *and* a bike, you'd also be in the union.

EXAMPLE 4 **UNION**

Find the union of the following sets.

(a) $A = \{5, 6, 2, 4\}$ and $B = \{1, 4\}$

(b) $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$

(c) $A = \mathbb{N}$ and $B = \varnothing$

Solution

(a) Again, the union of two sets is all the elements that are in either set (we only list repeated elements once):

$$\boxed{A \cup B = \{5, 6, 2, 4, 1\}}$$

Remember, the order in which we list the elements in a set is not significant.

(b) Here there are no elements common to both sets:

$$\boxed{A \cup B = \{a, b, c, d, x, y, z\}}$$

(c) Note that if you take the union of any set with the empty set, you'll get the set you started with, because the empty set doesn't add anything:

$$\boxed{A \cup B = \mathbb{N} \cup \varnothing = \mathbb{N}}$$

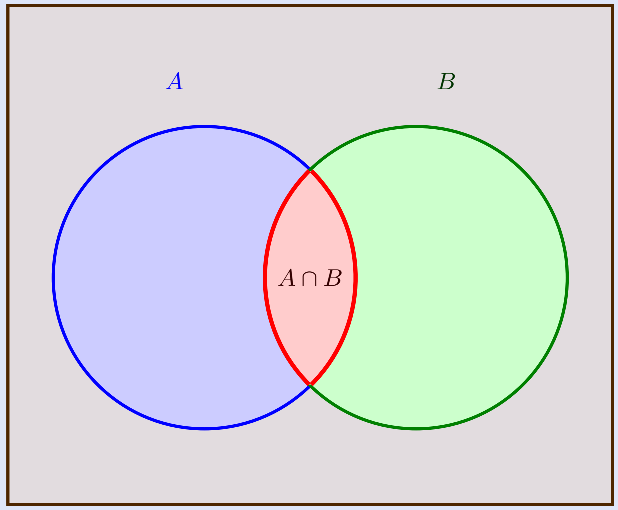
TRY IT

If $A = \{1, 3, 5, 7\}$ and $B = \{2, 3, 4\}$, find $A \cup B$.

Intersection

Intersection

The **intersection** of two sets A and B, denoted $A \cap B$, consists of all elements that are **in both** A **and** B.



$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

EXAMPLE 5 **INTERSECTION**

Find the intersection of each pair of sets.

(a) $\{1, 3, 5, 7, 9\} \cap \{2, 3, 4, 5\}$

(b) $\{x \mid x \text{ is even}\} \cap \{x \mid x \text{ is odd}\}$

(c) $\mathbb{N} \cap \varnothing$

Solution

(a) $\{1, 3, 5, 7, 9\} \cap \{2, 3, 4, 5\} = \boxed{\{3, 5\}}$

(b) $\{x \mid x \text{ is even}\} \cap \{x \mid x \text{ is odd}\} = \boxed{\varnothing}$

(c) $\mathbb{N} \cap \varnothing = \boxed{\varnothing}$

TRY IT If $A = \{1, 3, 5, 7\}$ and $B = \{2, 3, 4\}$, find $A \cap B$.

Note: look back at that last example and make sure you can verify that for any set A,

$$A \cup \varnothing = A$$
$$A \cap \varnothing = \varnothing$$

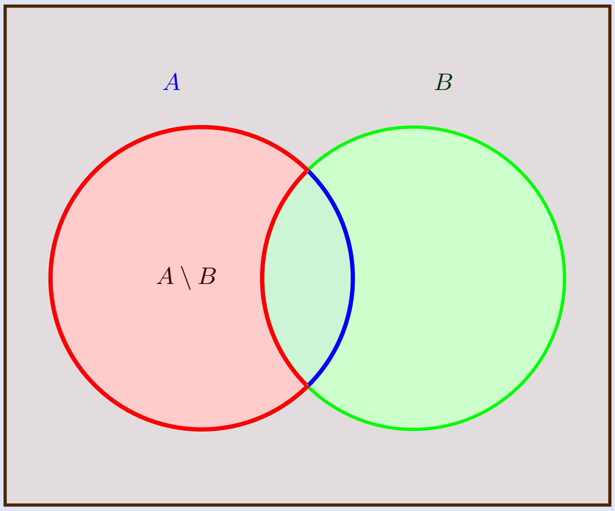
Difference

The difference between two sets is fairly intuitive: it consists of all the elements of the first set that are **not** elements of the second set. In other words, take the first set and *remove* any elements from it that also show up in the second set, and what's left is the difference.

The only unusual part is the symbol we use to denote this; it looks like a minus sign, but it's slanted to indicate that we're finding a *set* difference, not the difference between two numbers.

Difference

The **difference** of two sets A and B, denoted $A \setminus B$, consists of all elements that are **in A, but not in B.**



$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$

EXAMPLE 6 **DIFFERENCE**

If A consist of whole numbers from 1 to 9, and $B = \{7, 8, 9, 10, \ldots\}$, find $A \setminus B$.

Solution

Take away every element from A that also occurs in B, and you get

$$\boxed{A \setminus B = \{1, 2, 3, 4, 5, 6\}}$$

TRY IT

If $A = \{1, 3, 5, 7\}$ and $B = \{2, 3, 4\}$, find $A \setminus B$.

Note: Since the difference is everything that is in A AND (intersection) NOT (complement) in B, we can write the difference in terms of the intersection and complement:

$$A \setminus B = A \cap B^c$$

EXAMPLE 7 DIFFERENCE

Use the following sets to illustrate that

$$A \setminus B = A \cap B^c.$$

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$B = \{7, 8, 9, 10, \ldots\}$$

Solution We just found the difference $A \setminus B$ in the last example:

$$A \setminus B = \{1, 2, 3, 4, 5, 6\}$$

Now we just have to show that if we find $A \cap B^c$, we get the same answer:

$$B^c = \{\ldots, 4, 5, 6\} \longrightarrow A \cap B^c = \{1, 2, 3, 4, 5, 6\}$$

Combining Operations

As we just saw with $A \cap B^c$, we can use several set operations in combination. If we need to, we can use parentheses as grouping symbols to make the order of operations clear.

EXAMPLE 8 COMBINING OPERATIONS

Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7\}$, and $B = \{4, 5, 6, 7\}$. Find each of the following.

(a) $(A \cup B)^c$

(b) $A^c \cap B^c$

(c) $(A \cap B)^c$

(d) $A^c \cup B^c$

Solution

(a) Note that according to the parentheses, we need to start by finding $A \cup B$, and then we need to take the complement of this union:

$$A \cup B = \{1, 3, 4, 5, 6, 7\}$$
$$\Longrightarrow \boxed{(A \cup B)^c = \{2, 8, 9, 10\}}$$

(b) Since there are no parentheses, we'll first find the two complements individually, and then find their intersection:

$$A^c = \{2, 4, 6, 8, 9, 10\} \quad \text{and} \quad B^c = \{1, 2, 3, 8, 9, 10\}$$
$$\Longrightarrow \boxed{A^c \cap B^c = \{2, 8, 9, 10\}}$$

(c) Here again we start inside the parentheses:

$$A \cap B = \{5, 7\}$$
$$\Longrightarrow \boxed{(A \cap B)^c = \{1, 2, 3, 4, 6, 8, 9, 10\}}$$

(d) Finally, take the union of the complements:

$$A^c = \{2, 4, 6, 8, 9, 10\} \quad \text{and} \quad B^c = \{1, 2, 3, 8, 9, 10\}$$
$$\Longrightarrow \boxed{A^c \cup B^c = \{1, 2, 3, 4, 6, 8, 9, 10\}}$$

TRY IT

For the sets listed in the example above, find

(a) $A^c \cup B$

(b) $A \cap B^c$

(c) $B \cup B^c$

(d) $A \cap A^c$

Notice something interesting: that example, as a side note, illustrates the following two facts:

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

These are known as **De Morgan's laws**, and we'll restate and explain them in the next section, but before you get there, see if you can come up with an intuitive way of explaining why this makes sense. Maybe think of some simple, well-defined sets, and see if you can understand why the complement of the union is the intersection of the complements, and vice versa.

It may also help to draw some Venn diagrams to visualize this.

Using Three Sets

Extending the use of set operations to three sets (or more) is not difficult, as long as we're careful, especially with grouping.

SET OPERATIONS WITH THREE SETS — EXAMPLE 9

Suppose that H = {cat, dog, rabbit, mouse}, F = {dog, cow, duck, pig, rabbit}, and W = {duck, rabbit, deer, frog, mouse}.

(a) Find $(H \cap F) \cup W$.

(b) Find $(H \cap F)^c \cap W$.

Solution

(a) Start by finding the intersection: $H \cap F$ = {dog, rabbit}

Then take the union of this answer with W:

$$\boxed{(H \cap F) \cup W = \{\text{dog, duck, rabbit, deer, frog, mouse}\}}$$

(b) We already found this intersection: $H \cap F$ = {dog, rabbit}

Now we want elements that are NOT in this set, and ARE in W:

$$\boxed{(H \cap F)^c \cap W = \{\text{duck, deer, frog, mouse}\}}$$

TRY IT

For the sets listed in the example above, find

(a) $H \cup F^c \cup W$

(b) $H \cap (F \cup W)$

(c) $H^c \cap F \cap W$

If you want a bit of a challenge, try drawing a Venn diagram for the set $(H \cap F)^c \cap W$.

Summary of Set Operations

Complement: $A^c = \{x \mid x \in U \text{ and } x \notin A\}$

Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Intersection: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Difference: $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\} = A \cap B^c$

Exercises 7.2

In problems 1–28, let $U = \{1, 2, 3, \ldots, 10\}$, $A = \{1, 3, 5, 7\}$, $B = \{1, 2, 3, 4\}$, and $C = \{3, 4, 6, 7, 9\}$. Find each of the following sets.

1. A^c

2. B^c

3. $A \cup C$

4. $A \cap B$

5. $B \cap C$

6. $A \cup B$

7. $A \setminus B$

8. $C \setminus A$

9. $A^c \cap B^c$

10. $A^c \cup C$

11. $B \cup C^c$

12. $A \cap B^c$

13. $(A \cup C^c)^c$

14. $(B^c \cap C)^c$

15. $(A \cap B)^c$

16. $(B \cup A)^c$

17. $A \cup \varnothing$

18. $B \cup \varnothing$

19. $C \cup \varnothing$

20. $B \cap \varnothing$

21. $(A \cap B) \cup C$

22. $(A \cup C) \cap B$

23. $B \cup (A \cap C)$

24. $(A \cap B) \cup (C \cap B)$

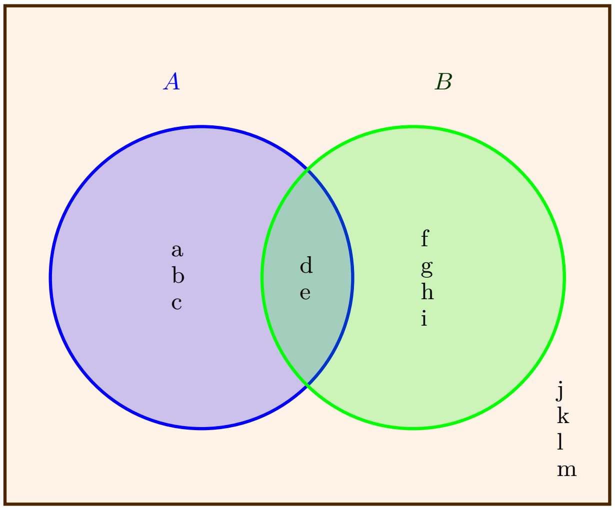
25. $(B \cup A) \cap (B \cup C)$

26. $(A \cup C)^c \cap B^c$

27. $A \cap B \cap C$

28. $A \cup B \cup C$

In problems 29–40, use the following Venn diagram to find each set.



29. A^c

30. $A \cup B$

31. $(A \cap B)^c$

32. $A^c \cup B^c$

33. $A \setminus B$

34. $B \setminus A$

35. $A^c \cap B^c$

36. $(A \cup B)^c$

37. U

38. $A^c \cup B$

39. $A \cap B^c$

40. $A \setminus U$

SECTION 7.3 Properties of Set Operations

In this section, we'll briefly state a few **identities**, including De Morgan's laws that we observed in the last section.

We won't see any rigorous proofs of these identities, but if you're curious, the way to prove that two sets A and B are equal is to show that every element in each of them is also in the other; in other words, to show that

$$A \subseteq B \quad \text{and} \quad B \subseteq A.$$

To do so, we would take an element from one set and show that it must belong to the other set, and then pick an element from the second set and show that it must belong to the first.

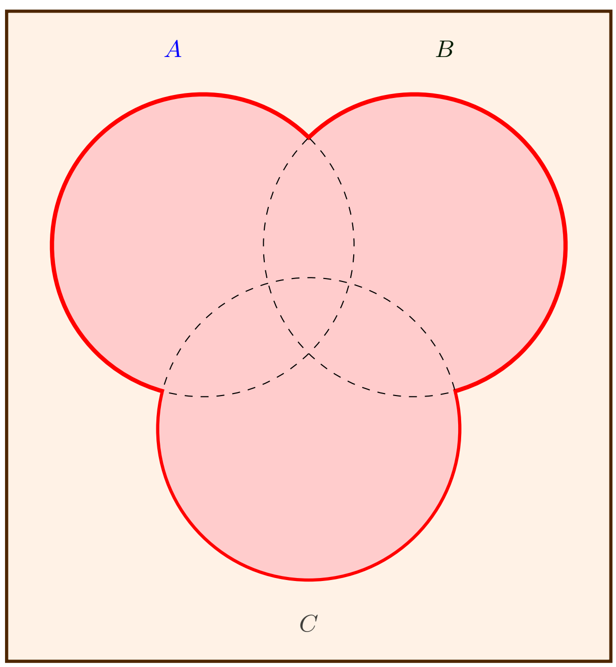
Instead of doing proofs like that, we'll simply illustrate each identity with an example and with an appropriate Venn diagram.

Associative Identities

The associative identities essentially state that the placement of parentheses doesn't matter when we're only doing one kind of operation (union or intersection):

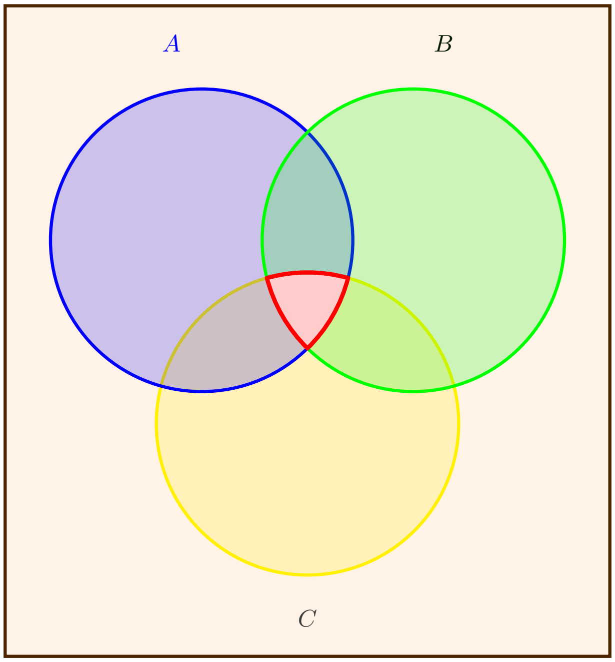
$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

Think about the first one, for instance, and let's see if we can make sense of it. It says that if we start with A, add in the elements of B, and then add in the elements of C, we'd get the same result as if we start with B, add in the elements of C, and finally add in the elements of A. Either way, we get the elements that are in any of the three sets.



$$(A \cup B) \cup C = A \cup (B \cup C)$$

The second is similar: whether you start by intersecting A and B, and then intersect that with C, or start by intersecting B and C, what you ultimately find is all the elements that belong to all three sets at once. For instance, if you start by intersecting A and B, you start with A, remove any elements that don't belong to B, and finally remove any elements that don't belong to C.



$$(A \cap B) \cap C = A \cap (B \cap C)$$

ASSOCIATIVE IDENTITIES EXAMPLE 1

Suppose we have the following sets:

$$A = \{1, 2, 3, 4, 5\}$$
$$B = \{2, 4, 6, 8, 9\}$$
$$C = \{1, 3, 9, 10, 11\}$$

Illustrate the associative identities with these sets.

(a) First, show that $(A \cup B) \cup C = A \cup (B \cup C)$ in this example:

$$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 9\}$$
$$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 11\}$$

$$B \cup C = \{1, 2, 3, 4, 6, 8, 9, 10, 11\}$$
$$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 11\}$$

(b) Then, show that $(A \cap B) \cap C = A \cap (B \cap C)$:

$$A \cap B = \{2, 4\}$$
$$(A \cap B) \cap C = \varnothing$$

$$B \cap C = \{9\}$$
$$A \cap (B \cap C) = \varnothing$$

In both cases, the associative identity held true.

Distributive Identities

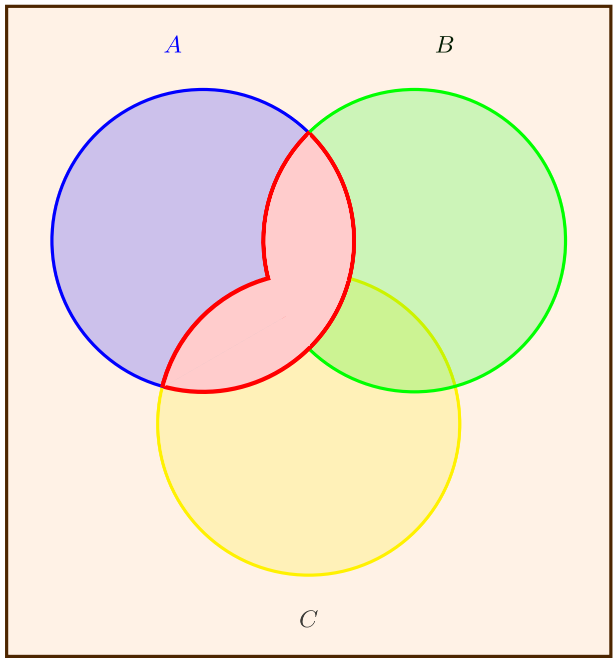
If you remember from your algebra classes, you can distribute something like

$$2(x+4) = 2x + 8$$

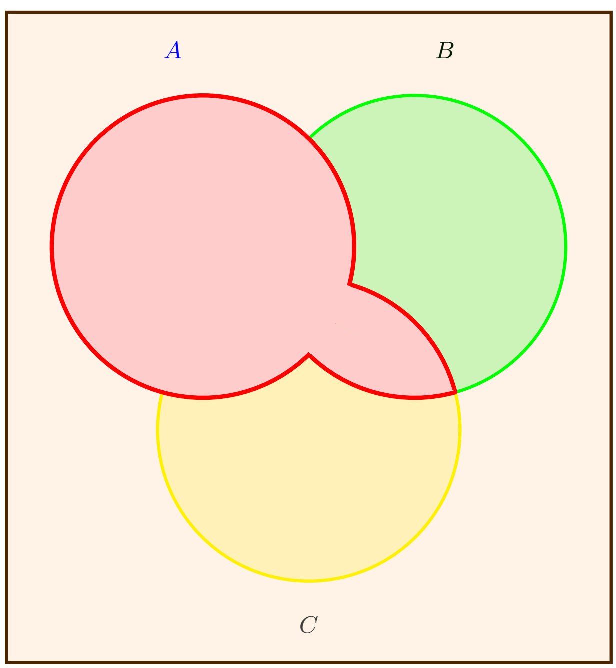
by applying the multiplication to both terms inside the parentheses. We can do something similar with set operations, where we can distribute one operation across parentheses where the other is used:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

The best way to make sense of these is probably to use the Venn diagrams below:



$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

DISTRIBUTIVE IDENTITIES — EXAMPLE 2

Suppose we have the following sets:

$$A = \{1, 2, 3, 4, 5\}$$
$$B = \{2, 4, 6, 8, 9\}$$
$$C = \{1, 3, 9, 10, 11\}$$

Illustrate the distributive identities with these sets.

(a) First, show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ in this example:

$$B \cup C = \{1, 2, 3, 4, 6, 8, 9, 10, 11\}$$
$$A \cap (B \cup C) = \{1, 2, 3, 4\}$$

$$A \cap B = \{2, 4\}$$
$$A \cap C = \{1, 3\}$$
$$(A \cap B) \cup (A \cap C) = \{1, 2, 3, 4\}$$

(b) Then, show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$:

$$B \cap C = \{9\}$$
$$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 9\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 9\}$$
$$A \cup C = \{1, 2, 3, 4, 5, 9, 10, 11\}$$
$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 9\}$$

In both cases, the distributive identity held true.

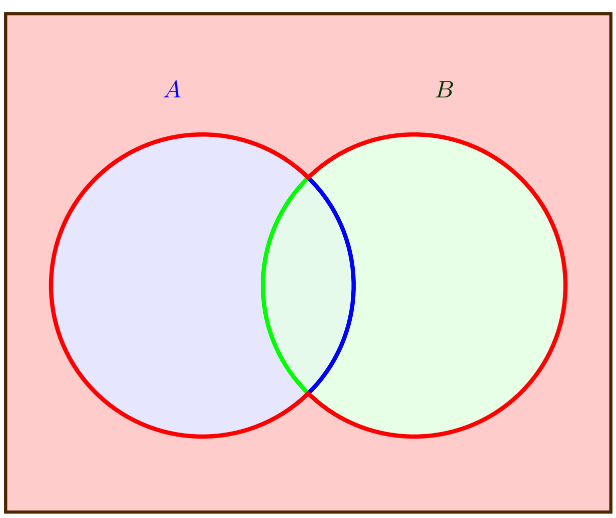
De Morgan's Laws

We've already noted these in passing. These are especially important when it comes to logic (for instance, in computer programming), which is closely linked with set theory.

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

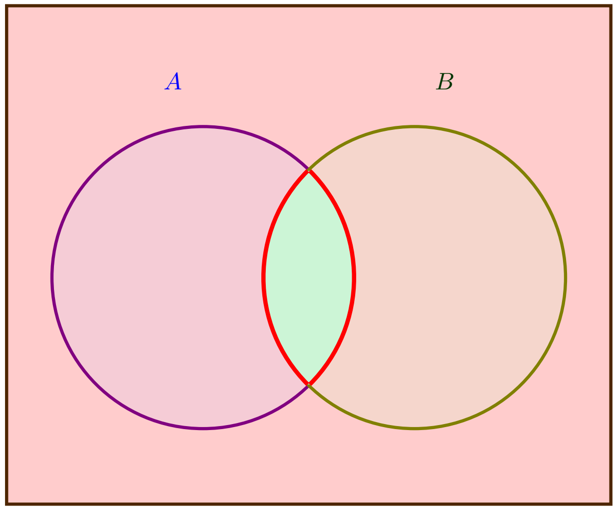
The first one basically says that the set of points that don't lie in either A or B is the same as the set of points that don't lie in A and don't lie in B.

The connection to logic is this: if you tell someone "don't get bread or milk," what you're really saying is "don't get bread, and don't get milk."



$$(A \cup B)^c = A^c \cap B^c$$

The second one says that points that don't lie in both A and B either don't lie in A or don't lie in B.



$(A \cap B)^c = A^c \cup B^c$

EXAMPLE 3 DE MORGAN'S LAWS

Suppose we have the following sets:

$$\begin{aligned} U &= \{a, b, c, d, e\} \\ A &= \{b, c\} \\ B &= \{a, c, e\} \end{aligned}$$

Illustrate De Morgan's laws with these sets.

(a) First, show that $(A \cup B)^c = A^c \cap B^c$ in this example:

$$\begin{aligned} A \cup B &= \{a, b, c, e\} \\ (A \cup B)^c &= \{d\} \end{aligned}$$

$$\begin{aligned} A^c &= \{a, d, e\} \\ B^c &= \{b, d\} \\ A^c \cap B^c &= \{d\} \end{aligned}$$

(b) Then, show that $(A \cap B)^c = A^c \cup B^c$:

$$\begin{aligned} A \cap B &= \{c\} \\ (A \cap B)^c &= \{a, b, d, e\} \end{aligned}$$

$$\begin{aligned} A^c &= \{a, d, e\} \\ B^c &= \{b, d\} \\ A^c \cup B^c &= \{a, b, d, e\} \end{aligned}$$

In both cases, De Morgan's laws held true.

Summary

Properties of Set Operations

Associative Identities

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

Distributive Identities

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

De Morgan's Laws

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

Exercises 7.3

Suppose that $U = \{1, 2, \ldots, 10\}$, $A = \{1, 3, 5, 7\}$, $B = \{2, 3, 4, 5\}$, and $C = \{4, 5, 6, 7, 8\}$.

Associative Identities

1. Show that $(A \cup B) \cup C = A \cup (B \cup C)$.

2. Show that $(A \cap B) \cap C = A \cap (B \cap C)$.

Distributive Identities

3. Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

4. Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

De Morgan's Laws

5. Show that $(A \cup B)^c = A^c \cap B^c$.

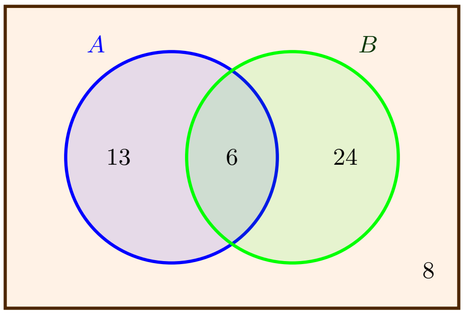
6. Show that $(A \cap B)^c = A^c \cup B^c$.

SECTION 7.4 Survey Problems

We can use sets to organize the results of a survey by visualizing the respondents with a Venn diagram. Specifically, if a survey asks more than one question, a Venn diagram can be a really handy tool.

MOVIE TYPES — EXAMPLE 1

Brianne surveyed students in her school to determine what types of movies they prefer: science fiction, comedy, both types of movies, or neither type of movie. If A represents science fiction and B represents comedy, the following diagram shows the result of her survey.



Answer the following questions.

(a) How many students did Brianne survey?

(b) How many students like science fiction?

(c) How many students like **only** science fiction?

(d) How many students like comedy?

(e) How many students like comedy, but not science fiction?

(f) How many like both science fiction and comedy?

(g) How many like science fiction or comedy?

(h) How many like neither?

(i) How many do not like science fiction?

(j) How many do not like comedy?

Solution

(a) How many students did Brianne survey?

This includes everyone in the universal set; the number of people in this set is the sum of all the numbers that are shown:

$$13 + 6 + 24 + 8 = \boxed{51}$$

Therefore, she surveyed a total of 51 students.

(b) How many students like science fiction?

The total number of students who like science fiction is all those that are in the blue circle:

$$13 + 6 = \boxed{19}$$

There are 19 students who like science fiction.

(c) How many students like **only** science fiction?

This is all those in the blue circle that are NOT also in the green circle, for a total of $\boxed{13}$

(d) How many students like comedy?

Similarly, there are

$$24 + 6 = \boxed{30}$$

students who like comedy.

(e) How many students like comedy, but not science fiction?

This is all those in the green circle but outside the blue circle; there are $\boxed{24}$ of these students.

(f) How many like both science fiction and comedy?

Those who like both lie in the intersection of the two sets; there are $\boxed{6}$ of these.

(g) How many like science fiction or comedy?

The word OR indicates that this represents the union of these two sets: there are

$$13 + 6 + 24 = \boxed{43}$$

students who like science fiction or comedy.

(h) How many like neither?

This would be the $\boxed{8}$ students outside of both circles.

(i) How many do not like science fiction?

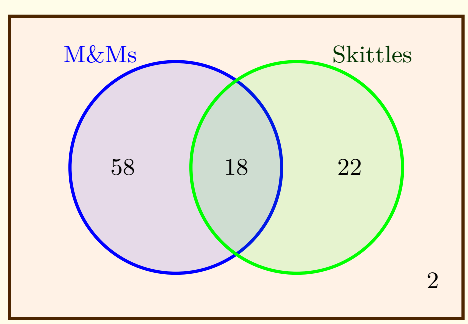
There are two ways to do this. On the one hand, we could add up all the numbers outside the blue circle. On the other, we could subtract those who like science fiction (from part b) from the total number who were surveyed (from part a). Either way, we find that there are $\boxed{32}$ students who do not like science fiction.

(j) How many do not like comedy?

We can solve this one in a similar way; there are $\boxed{21}$ students who do not like comedy.

TRY IT

A survey asked 100 people two questions: "Do you like M&Ms?" and "Do you like Skittles?" The results are shown below.



(a) How many people liked M&Ms, but not Skittles?

(b) How many people liked Skittles?

(c) How many people like M&Ms or Skittles?

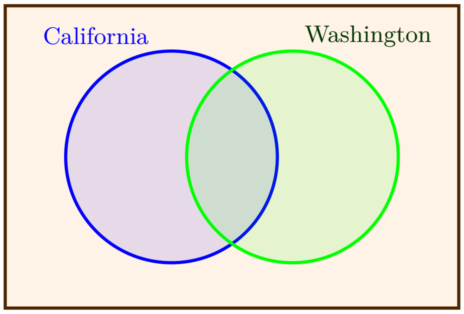
If the results of a survey are not given as a Venn diagram, we can still build one to visualize the results.

TRAVEL DESTINATIONS — EXAMPLE 2

Of the students in Latoya's class, eight have been to California, nine have been to Washington, and five have been to both California and Washington. How many students have been to California or Washington?

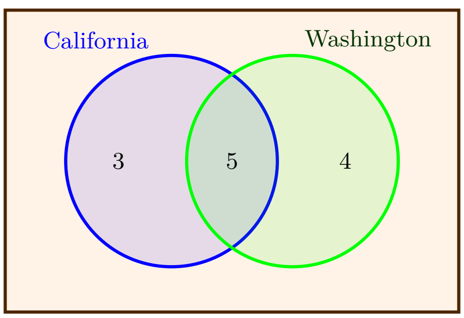
Solution

We can draw a diagram like in the previous example to answer this question:



Here's how to fill this in: **start with the innermost intersection**. The overlap represents those who have been to both states, of which there are five.

Then, we know that a total of eight students have been to California; we've already accounted for five of them in the overlap, so there are three students in the left circle but NOT in the right circle. Similarly, there are a total of nine students who have been to California, of which five have already been accounted for, leaving four others in the right circle.



Finally, we can answer the question: there are a total of 12 students (in the union of the two sets) who have been to either California or Washington.

TRY IT

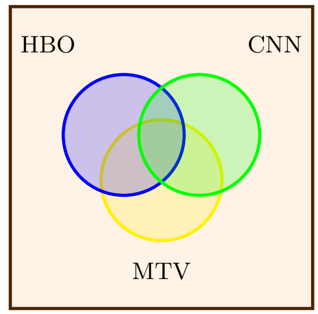
Of the children in Sofia's class, seven like to use markers, five like to use colored pencils, and three like to use both. How many children like to use colored pencils but not markers?

In examples like that one with only two categories, we could potentially answer the question without drawing a diagram, but drawing the diagram becomes more and more helpful for more complicated questions, like those with three categories.

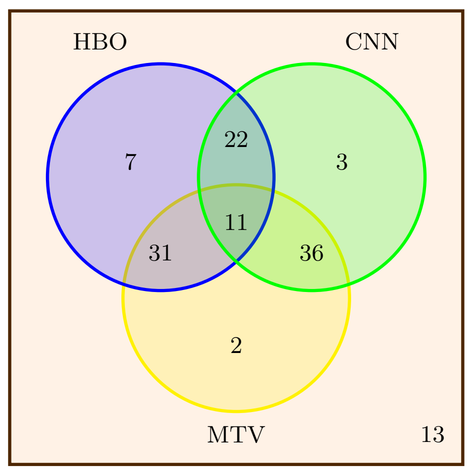
EXAMPLE 3 TV NETWORKS

A survey of 125 people was conducted to determine the popularity of HBO, CNN, and MTV. The survey found that 71 people watch HBO, 72 watch CNN, and 80 watch MTV. Furthermore, 33 watch HBO and CNN, 42 watch HBO and MTV, and 47 watch CNN and MTV. Finally, 11 watch all three.

Fill in the Venn diagram below.



Solution As before, start with the innermost intersection and work outwards. There are 11 in the center, and a total of 33 in the intersection of HBO and CNN, so there are 22 in the intersection of HBO and CNN, but outside the center. Go on and fill in the other two intersections, and then subtract to fill in the rest of the circles.



Now that we have the Venn diagram, we can answer questions like the following ones.

(a) How many watch only MTV?

2

(b) How many watch CNN and MTV, but not HBO?

36

(c) How many do not watch any of these networks?

13

(d) How many do not watch CNN?

53

(e) How many watch HBO or MTV?

109

TRY IT

Fifty students were surveyed and asked if they were taking a social science, humanities, or natural science course the next semester.

The survey found that 21 were taking a social science course, 26 were taking humanities, 19 were taking natural science. Also, 9 were taking social science and humanities, 7 were taking social science and natural science, and 10 were taking humanities and natural science. Finally, 3 students were taking all three.

(a) How many students are taking only a social science course?

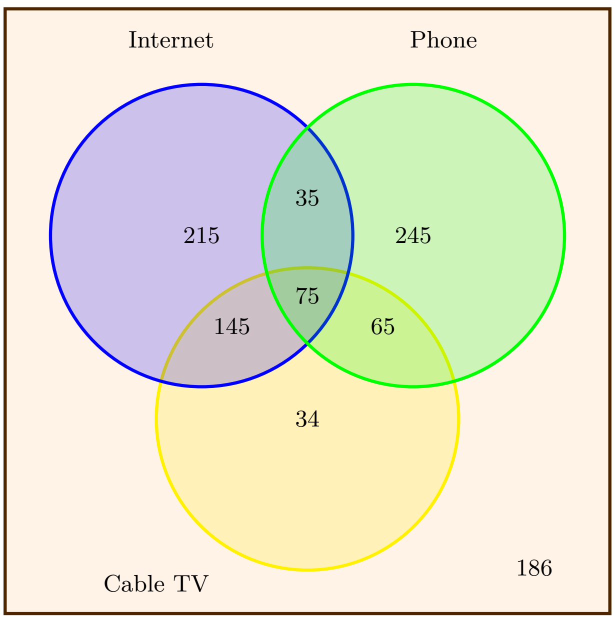
(b) How many are taking natural science and social science, but not humanities?

(c) How many are taking none of these courses?

COMCAST SERVICES

EXAMPLE 4

A survey of 1000 households found that 470 use Comcast internet service, 420 use their telephone service, and 319 use their cable television. Of these, 140 families use the telephone and television services, 220 families use the internet and television service, and 110 use the internet and telephone. There are 75 families who use all three. The diagram below summarizes this information.



1. How many households in this survey do not use any of these services?

   186

2. How many use exactly one of these services?

   These are all the ones that lie in only one circle:

   494

3. How many use exactly two of these services?

   These are all the ones that lie in the intersection of two circles, but not in the very center:

   245

Exercises 7.4

1. A survey asked 200 people what beverage they drink in the morning, and offered two possible choices: tea and coffee. Suppose that 20 answered tea only, 80 answered coffee only, and 40 answered both.

(a) How many people drink tea in the morning?

(b) How many people drink neither tea nor coffee?

2. A survey asked 100 people whether they used Twitter or Facebook in the last month. Of those surveyed, a total of 40 used Twitter, 70 used Facebook, and 20 used both.

(a) How many people used only Facebook?

(b) How many people used neither Facebook nor Twitter?

3. Out of 100 customers of Domino's Pizza, 60 ordered pizza with onions and pepperoni, 80 ordered it with pepperoni, and 72 ordered it with onions.

(a) How many ordered onions but not pepperoni?

(b) How many ordered pepperoni but not onions?

(c) How many ordered neither onions nor pepperoni?

4. Out of 100 students surveyed, 24 rent movies, 20 rent movies and go the theater, and 15 do neither.

(a) How many students only rent movies?

(b) How many students only go to the theater?

(c) How many students go to the theater or rent movies?

5. An independent survey agency was hired by the Metro to find out how many people commute to their school or job. The agency interviewed 1000 commuters and submitted the following report:

631 came by car	373: car and bus
554 came by bus	301: bus and metro
759 came by metro	268: car and metro

231: all three types of transportation

The Metro refused to accept the report, stating that it was inaccurate. Why?

6. One hundred fifty people were surveyed and asked if they believed in UFOs, ghosts, and Bigfoot.

43 believed in UFOs	8: ghosts and Bigfoot
25 believed in Bigfoot	10: UFOs and ghosts
44 believed in ghosts	5: UFOs and Bigfoot

2 believed in all three

(a) How many people surveyed believed in at least one of these things?

(b) How many people believed in ghosts and Bigfoot, but not UFOs?

(c) How many people didn't believe in any of the three?

(d) How many people believed in Bigfoot only?

7. A survey asked students whether they had seen *Star Wars*, *The Matrix*, or *Lord of the Rings (LotR)*.

24 had seen *Star Wars*	10: *Star Wars* and *The Matrix*
18 had seen *The Matrix*	12: *The Matrix* and *LotR*
20 had seen *LotR*	14: *Star Wars* and *LotR*

6 had seen all three

(a) How many students have seen exactly one of these movies?

(b) How many students have seen only *Star Wars*?

(c) How many students have seen *Star Wars*, but not *LotR*?

(d) How many students have not seen *The Matrix*?

8. A survey was given asking whether respondents watch movies at home from Netflix, Redbox, or Amazon Video.

53 only use Netflix	48: only Netflix and Redbox
62 only use Redbox	16: only Redbox and Amazon
24 only use Amazon Video	30: only Netflix and Amazon
10 use all three	25: none of these

(a) How many people use Redbox?

(b) How many people use at least one of these?

(c) How many people were surveyed?

9. A survey asked buyers whether color, size, or brand influenced their choice of cell phone. The results are below.

5 only said color	20: only color and size
8 only said size	53: only size and brand
16 only said brand	42: only color and brand
102 said all three	20: none of these

(a) How many people were influenced by brand?

(b) How many people were influenced by color or size?

(c) How many people were surveyed?